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Time dependent Ginzburg-Landau equations for modeling vortices dynamics in type-II superconductors with defects under a transport current

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Abstract

In the present work, dynamics of vortex structure in the type-II superconductors with defects under a transport current has been studied using the time dependent Ginzburg-Landau equations. The vortex interactions with the pining centers in two-dimensional cases have been investigated. The model has been solved using ψU-method and finite element method. The defects are normal regions and it size was greater-than through influence of proximity effect. In our simulation, we have considered that using model agree well for description of vortices behavior in inhomogeneous sample under a transport current. We have considered defects influence and relative position on dynamics of vortices. Also we have calculated the values of critical current density at various configuration of sample.

Keywords: Keywords: superconductivity, Ginzburg-Landau equations, pinning, vortices

1. Introduction

Below a certain field, the first critical field H_{c1}, the type-II superconductor is still a perfect diamagnet, but in fields just above H_{c1} magnetic flux does penetrate the material. It is concentrated in well separated "vortices" of size λ, the magnetic penetration depth, carrying one unit of flux. Vortices can move as a result of interaction with the transport current. Vortex motion in superconductors gives rise to an electric field. Vortex motion thus produces dissipation in a superconductors which thus posesses a finity resistivity. This is why it is very important to study how vortices can be made immobile (pinned). After the discovery of high-temperature superconductors study vortex states has been attracting more attention among scientists around the world. One approach to solving this problem is to model based on time dependent Ginzburg-Landau (GL) equations. To model the mixed state of superconductors, this approach is currently the most correct and accurate, but its implementation requires large computational power. Most modeling is part of the dimensionless time of the GL equations and their modifications [1], [2], [3], [4].

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2. Time dependent GL model for superconductivity

Time dependent Ginzburg-Landau equations are a system of nonlinear partial differential equations for the order parameter ψ and vector potential $\vec{A}$:

$$\frac{1}{D}(\frac{\partial}{\partial t} + i\frac{2e\varphi}{\hbar})\psi = -(\frac{\nabla}{i} - \frac{2e}{\hbar c}\vec{A})^2 - \frac{1}{\xi^2(T)}(|\psi|^2 - 1)\psi + f(\vec{r}, t) \qquad in\ \Omega \tag{1}$$

$$\vec{j} = \sigma(-\nabla\varphi - \frac{1}{c}\frac{\partial\vec{A}}{\partial t}) + Re(\psi^*(\frac{\nabla}{i} - \frac{2e}{\hbar c}\vec{A})\psi)\frac{\hbar c^2}{8\pi e\lambda^2(T)} \qquad in\ \Omega \tag{2}$$

$$\vec{n}(i\hbar\nabla + \frac{e}{c}\vec{A})\psi = 0 \qquad on\ \partial\Omega \tag{3}$$

where D is a phenomenological diffusion constant, and φ is the electric potential, , included to retain the gauge invariance of the equations, f is the thermal fluctuations, Ω is the domain occupied by the superconducting material, $\vec{n}$ is the unit outward normal to $\partial\Omega$. The constants in the equations are $\hbar$, Plancks constant divided by 2π; c, the speed of light; e , the charge of electron; σ, the electrical conductivity; $\xi(T)$, the coherence length; $\lambda(T)$, the characteristic length scale for variations of the magnetic field. As usual, i is the imaginary unit, "*" denotes complex conjugation and "*Re*" denotes real part of complex number.

2.1. Dimensionless form of time dependent GL equations and Gauge choice

We believe that the temperature dependence of the coherence length and penetration depth of the magnetic field have the form: $\xi(T) = \frac{\xi(0)}{(1-T)^{1/2}}$, $\lambda(T) = \frac{\lambda(0)}{(1-T)^{1/2}}$, $T = \frac{T}{T_c}$. We scale length in multiples of the coherence length $\xi(0)$. $\kappa = \frac{\lambda}{\xi}$ is the GL parameter. In scaled units equations (1), (2), (3) become:

$$(\frac{\partial}{\partial t} + i\Phi)\psi = -\frac{1}{\eta}[(-i\nabla - \vec{A})^2\psi + (1 - T)(|\psi|^2 - 1)\psi] + f(\vec{r}, t) \qquad in\ \Omega \tag{4}$$

$$\kappa^2\nabla \times \nabla \times \vec{A} = (1 - T)Re[\psi^*(-i\nabla - \vec{A})\psi] + [-\frac{\partial\vec{A}}{\partial t} - \nabla\Phi] \qquad in\ \Omega \tag{5}$$

$$\vec{n}(-i\nabla - \vec{A})\psi = 0 \qquad on\ \partial\Omega \tag{6}$$

where η is positive value, which determines the ratio of the characteristic relaxation times for ψ and $\vec{A}$. $\kappa < 1/\sqrt{2}$ describes a type-I superconductor, $\kappa > 1/\sqrt{2}$ describes a type-II superconductor.

The nondimensional time dependent GL equations are invariant under a gauge transformation:

$$\vec{A} = \vec{A}' + \nabla\theta, \qquad \psi = \psi' \exp(i\theta), \qquad \Phi = \Phi' - \frac{\partial\theta}{\partial t} \tag{7}$$

where θ can be any real scalar-valued function of position and time.

For the calculations we used zero electric potential gauge $\Phi = 0$.

The partial differential equations (4) and (5) are solved numerically in the two dimensional space with boundary condition (6) for external applied magnetic field $\vec{H} = (0, 0, H_{ext})$. The initial conditions are $|\psi| = 1$ and zero magnetic field inside the superconductor. Transport current is introduced similar to the work [5]. In our numerical calculations the pinning centers (defects) are normal inclusions. To the boundary of a superconductor with normal metal used equations and boundary conditions of work [6].

2.2. Numerical methods

Generally, for the numerical simulations time dependent GL equations are used finite-difference methods [7] and the finite-element method [8]. In our work we used both methods.

It is crucial that the numerical approximation does not depend on the particular choice of gauge. The problems of gauge invariance solved by introducing the new complex variables U_x, U_y instead of the vector potential:

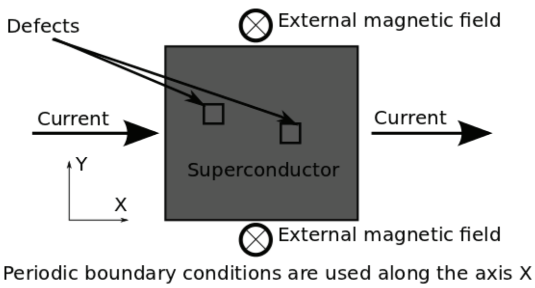
$$U_x(x, y, t) = \exp(-i \int_{x_0}^{x} A_x(\xi, y, t) d\xi) \tag{8}$$

$$U_y(x, y, t) = \exp(-i \int_{y_0}^{y} A_y(x, \xi, t) d\xi) \tag{9}$$

3. Results

We have considered the problem of the flow of transport current in type-II superconductors with a normal metal inclusions. Normal metal inclusions act as pinning centers. In the present simulation, the GL parameter κ is assumed to be 2. Periodic boundary conditions are used on boundaries perpendicular to a transport current.

Here, we show the simulation results. First, the pinning centers are the inclusions of a normal metal square with sides of 1.5ξ (because of the proximity effect). And they were uniformly distributed to various concentrations. Problem definition is presented on fig.1 (left). Figure 1 (right) is critical current density as function of pinning centers density. From this figure it is visible that the maximum critical current density is observed at concentration of the pinning centers 0.17.



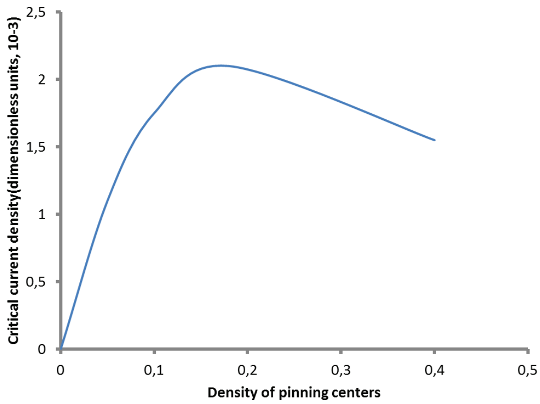


Fig. 1. Left: Problem definition, pinning vortices on normal inclusions (square with sides of 1.5ξ). Normal inclusions are uniformly distributed. Right: Critical current density as function of pinning centers density.

Further, the pinning center is a layer of normal metal between two superconductors. Problem definition is presented on fig.2 (left). Figure 2 (right) is critical current density as function of width of normal metal layer between two superconductors.

The maximum critical current density is observed at width of normal metal layer between two superconductors 3.5ξ, see fig.2 (right). The problem about pinning vortices on a metal layer between two superconductors in time has been considered. Figure 3 is spatial distribution of order parameter at some moments of time (top) and voltage as function of time (bottom). At the initial moment of penetration of vortical structure speed of its movement is maximum, and consequently voltage is maximum (fig.3 bottom, a point 1). Vortices attract to boundary of a metal layer and their speed increases. Next vortices pinned on a metal layer and on the boundary of metal layer appears supercurrent. This supercurrent creates a force repelling

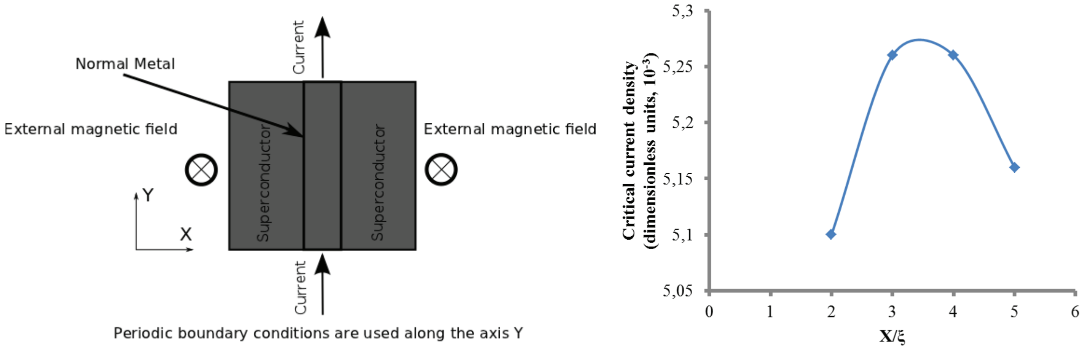


Fig. 2. Left: Problem definition, pinning vortices on layer of normal metal between two superconductors. Right: Critical current density as function of width of normal metal layer between two superconductors.

vortices. The number of vortices pinned on a metal layer increases. In some moment of time the force from supercurrent is very increasing that lead to stopping the motion of vortex structure and deacreasing voltage to zero.

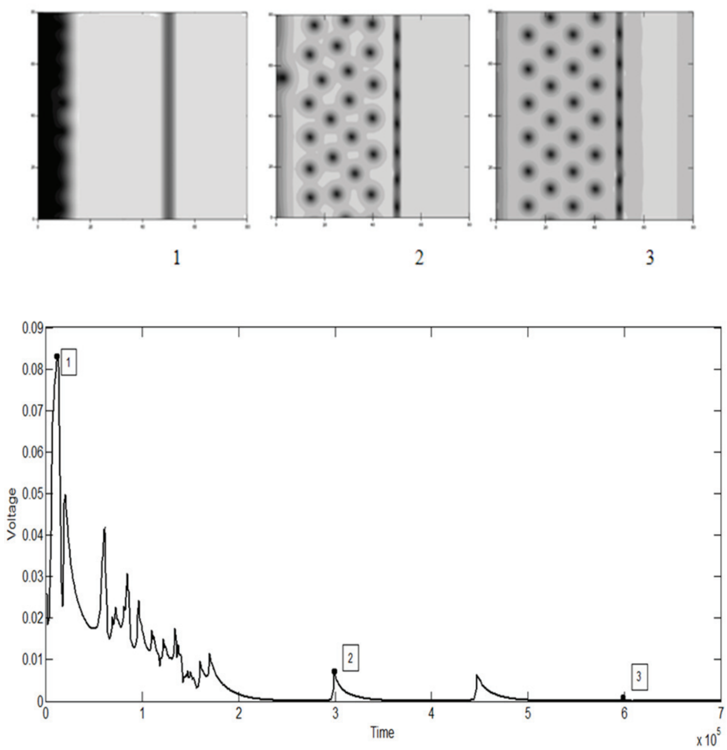


Fig. 3. Top: Spatial distribution of order parameter at some moments of time. Bottom: Voltage as function of time.

4. Conclusion

The phenomenon of pinning of the vortex structure on the system of "point" defects (inclusions of a normal metal square with a side of 1.5ξ) distributed uniformly investigated. The most critical current is observed at a concentration of pinning centers 0.17. The phenomenon of vortex pinning at the metal interlayer between two superconductors investigated. The optimal (in terms of maximum critical current) width of metal layer between two superconductors is found, its 3.5ξ. From a comparison of calculations by pinning of vortices in a system of point defects distributed uniformly and pinning of vortices on a metallic interlayer between two superconductors can be seen that the second way of pinning more efficient (in terms of increasing the critical current) than first.

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